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THE UNIVERSITY OF MICHIGAN
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TIME-DELAY ANALYSIS FOR PACKET-SWITCHING

COMMUNICATIONS NETWORKS

by

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Abstract

Mean delay times at certain superposition nodes are found for messages of variable length in a packet-switched network. These improve on some previous approximations. The output from these nodes is such that exact mean delays in some larger networks may be found.

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A system of queues with deterministic servers has been proposed as a model for a packet switched message network. One advantage of constant service times is that the distribution of the total delay time for a set of servers in series can be determined (see Avi-Itzhak [1]). This has been extended to variable message length systems by Rubin [2]. In a network, however, the flow in a series of such service facilities will be subject to interference, either by externally generated messages, assumed to form a Poisson process, or by the output of other sections of the network.

Rubin [3] has suggested some approximations for the mean delay times in simple superpositions of this type, based on considering the system as seen by each type of traffic separately. The approximations require Poisson assumptions at each node. This note shows that for the general class of systems in which delays are large, exact expressions for the mean delay times may be found, which allow for messages of varying lengths, and further that the combined output from such systems is often of a form which gives exact results for the delays at subsequent stations. In Figure 1, for example;

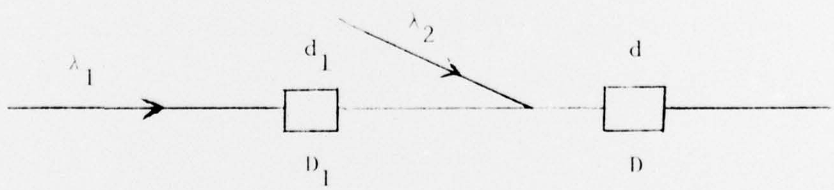

Figure 1

where the input streams are Poisson with parameters λ_1, λ_2, and the two stations, D_1 and D, have constant processing times, d_1, d respectively, Rubin's analysis gives the mean delay of a packet at D as:

$$W = \frac{1}{2}\frac{(\lambda_1+\lambda_2)d^2}{1-(\lambda_1+\lambda_2)d} - \frac{(1-\lambda_2 d)}{2}\frac{\lambda_1 d_1^{\,2}}{1-\lambda_1 d_1}, \quad d_1 < \frac{d}{1-\lambda_2 d}$$

$$\frac{1}{2}\frac{\lambda_2 d^2}{1-(\lambda_1+\lambda_2)d}, \quad d_1 \geq \frac{d}{1-\lambda_2 d}.$$

An exact result will be shown to be

$$W = \frac{1}{2}\frac{(\lambda_1+\lambda_2)d^2}{1-(\lambda_1+\lambda_2)d} - \frac{\lambda_1^{\,2} d_1^{\,2}}{2(\lambda_1+\lambda_2)(1-\lambda_1 d_1)}, \quad d_1 \leq d. \qquad \text{(Corollary 3.1)}$$

Furthermore, in this case the output from D is identical to that from an M/D/1 queue, and so the mean delay at any subsequent station may be found exactly.

Generally, messages arrive as a set of n independent Poisson processes at n stations. λ_i^{-1} is the mean interarrival time at the i-th station. Message lengths are independent and take up an integral number of packets, each taking constant processing time. The combined output of the n stations then queues for service at station D. Messages are processed at D in order of arrival, with each packet taking time d. The system can be modelled as a set of queues with group arrivals, with b(i,j) the probability that a group of size j arrives at server i, whose service time is d_i.

Let

$$B(j) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} b(i,j), \quad j=1,2,\ldots, \quad \lambda=\lambda_1+\ldots+\lambda_n,$$

$$\mu = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda}\mu_i, \quad \sigma^2 = \sum_{i=1}^{n}\left(\frac{\lambda_i}{\lambda}\right)^2 \sigma_i^{\,2},$$

where μ_i and $\sigma_i^{\,2}$ are the mean and variance of the group size in the arrival

stream to station i.

Theorem 1. For $0 \le d_i \le d$, $i=1,\dots,n$, and $\lambda\mu d < 1$, the output from D is identical to that of a group arrival M/D/1 queue whose mean interarrival time is λ^{-1}, service time is d, and group size distribution is B(j).

Proof. We show that the idle time and busy period distributions at D are those of an M/D/1 queue.

Note that since $d_i \le d$ all arrivals from a particular busy period at server i must be served in the same busy period at D, and hence that an arrival from server i who initiates a busy period at D must also have initiated a busy period at server i. Also the superposition of the n independent arrival processes is still a Poisson process with group arrivals. The group size distribution is now B(j).

Suppose that D is about to become idle at some time t. An arrival at i after $t-d_i$ will be served during the next busy period at D. If this arrival occurs at time $t-d_i + x$ then by the note above it will arrive at D at time $t + x$.

Thus the idle time at D is independent of the type of the last arrival, and has a distribution function given by:

$$\int_0^t \lambda_1 e^{-\lambda_1 x} e^{-(\lambda_2+\dots+\lambda_n)x}\,dx + \dots\dots + \int_0^t \lambda_n e^{-\lambda_n x} e^{-(\lambda_1+\dots+\lambda_{n-1})x}\,dx$$

$$= 1 - e^{-\lambda t} .$$

We write $P(i,n|m)$ for probability that a busy period is initiated by an arrival from stream i and is n service periods long, given that the first arrival

came from a group of m. If the busy period appears as if it will end at time t, again an arrival at server i will only extend it if the arrival occurs time t - d_i. Thus from the independence of the arrival streams

$$P(i,m|m) = \frac{\lambda_i}{\lambda} e^{-\lambda_i md} e^{-(\lambda_1+\ldots+\lambda_{i-1}+\lambda_i+\ldots+\lambda_n)md} ,$$

$$= \frac{\lambda_i}{\lambda} e^{-m\lambda d} .$$

Also by conditioning on the completion time of the first m service periods

$$P(i,j+m|m) = \sum_{r=1}^{j} \frac{\lambda_i}{\lambda} e^{-m\lambda d} \sum_{k=1}^{r} \frac{(m\lambda d)^k}{k!} b^k(r) P(j|r),$$

since as far as the busy period distribution is concerned there is no difference between a busy period initiated by an arrival requiring r service periods, and one initiated by a queue of r customers. Here $P(j|r) = \sum_{i=1}^{n} P(i,j|r)$, and $b^k(r)$ is the probability that the total number of arrivals in k groups is r. $b^k(r)$ is the coefficient of z^r in $(\sum_{j=1}^{\infty} B(j)z^j)^k$.

$$P(m|m) = e^{-m\lambda d}, \quad m=1,2,\ldots$$

$$P(j+m|m) = \sum_{r=1}^{j} e^{-m\lambda d} \sum_{k=1}^{r} \frac{(m\lambda d)^k}{k!} b^k(r) P(j|r). \tag{1}$$

The equations (1) are identical to those for the length of a busy period of an M/D/1 group arrival queue. Their solution is the Borel-Tanner distribution (see Bhat [4], page 10, for example),

$$P(j|m) = \frac{m}{j} e^{-j\lambda d} \sum_{k=0}^{m-j} \frac{(j\lambda d)^k}{k!} b^k(j-m), \; j \geq m.$$

Since the busy period length is conditionally independent of the source of the first arrival, the output from D consists of a sequence of busy periods whose probability functions are given by $P(j|m)$, where m has the distribution $B(m)$, separated by independent exponentially distributed idle times.

The mean delays at any subsequent set of stations in series can now be found from [1] and [2].

Since we have shown in Theorem 1 that the output from such a system is identical to that from a M/D/1 queue it seems reasonable to expect that the two mean waiting times are related. Although we can now calculate the mean time to the n-th departure from the system it is not necessarily true that the arrival who is the n-th departure will have been the n-th arrival to the system. We need to alter the service discipline at server D in order to be able to compare mean arrival and departure times.

A particular order of service is said to be work-conserving if the server is not idle when the queue is non-empty, and each customer receives his full service requirement upon entering service. Since the queue length distributions under work-conserving orders will be identical, from Little's formula we have:

Lemma 1: For a single server queue with a work-conserving order of service, the mean waiting time of an arbitrary customer is the same as if the order or service is first-come-first-served.

We modify the system as follows. The arrival process to the i-th server starts at time $(0-d_i)$. At one of the starting times an arrival occurs so with probability one there is an arrival to D at time zero. If an

arrival occurs to the i-th server at time t then it is numbered in the sequence of arrivals as if it had occurred at time $t+d_i$. On arrival at D groups are served in order. It can be seen that since all arrivals during a single busy period are served in the same busy period at D the modified system is work-conserving. The following result allows us to identify the time of the n-th departure.

Theorem 2: In the modified system the departure stream from D is the same as that from an M/D/1 group arrival queue.

Proof: The proof of this follows the same set of steps and produces the same expressions as that of Theorem 1.

The mean message delay at D, W, can now be found. W is the time from the arrival of the first packet in a message until processing begins.

Theorem 3: For $d_i \le d$, $\lambda\mu d < 1$,

$$W = \frac{\lambda d^2}{2(1-\lambda\mu d)}(\mu^2+\sigma^2) - \sum_{i=1}^{n} \frac{(\lambda_i d_i)^2}{2\lambda(1-\lambda_i\mu_i d_i)}(\mu_i^2+\sigma_i^2).$$

Proof: By applying the Pollaczek-Khintchine formula to an M/G/1 queue with a discrete service time distribution, for large n the departure mean time of the first packet of the n-th message is

$$T_n = \frac{n-1}{\lambda} + d + \frac{\lambda d^2}{2(1-\lambda\mu d)}(\mu^2+\sigma^2).$$

For the modified system the arrival time of this packet at the station D is

$$A_n = \frac{n-1}{\lambda} + \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} \frac{\lambda_i d_i^2}{2(1-\lambda_i\mu_i d_i)}\left(\mu_i^2+\sigma_i^2\right).$$

Since $W = \lim_{n\to\infty}(T_n - A_n - d)$ the result follows.

Corollary 3.1. If each message consists of one packet, then

$$W = \frac{\lambda d^2}{2(1-\lambda d)} - \sum_{i=1}^{n} \frac{(\lambda_i d_i)^2}{2\lambda(1-\lambda_i d_i)}.$$

Corollary 3.2. The probability that a message experiences no delay in the system is identical for each type of traffic and is given by $1-\lambda\mu d$.

Proof: Messages initiating a busy period at D will necessarily be undelayed. From Theorem 1 the type of such a message is random and so the required probability is the same as the probability of not waiting for an arbitrary customer in an M/D/1 queue.

If each of the n input streams to D is identical, (i.e., $\lambda_1 = \ldots = \lambda_n$, $d_1 = \ldots = d_n$, $b(i,j)=B(j)$), the mean delay of the i-th stream at D is

$$W_i = \frac{\lambda d^2}{2(1-\lambda\mu d)} (\mu^2 + \sigma^2) - \frac{\lambda_i d_i^2}{2(1-\lambda_i \mu_i d_i)} (\mu_i^2 + \sigma_i^2). \tag{2}$$

We conjecture that (2) does not generalize to various input streams.

Since the group size distribution is known the waiting time of individual packets at D can also be found.

Theorem 1 does not cover the case where $d_i > d$ for some i. Since some interdeparture intervals will be of length d_i while others are of length d, it is clear that the output from D cannot be identified with that from an M/D/1 queue. In practical situations this defect may not be important for the following reasons. If most of $d_1, \ldots, d_n$ are greater than d then mean waiting time at D will be small and so Rubin's approximations, giving a tight lower bound, may be appropriate. In those situations where long waiting times are likely to occur, that is a station giving long processing times in series with D, the mean delay can be determined without anlayzing the output of D as follows.

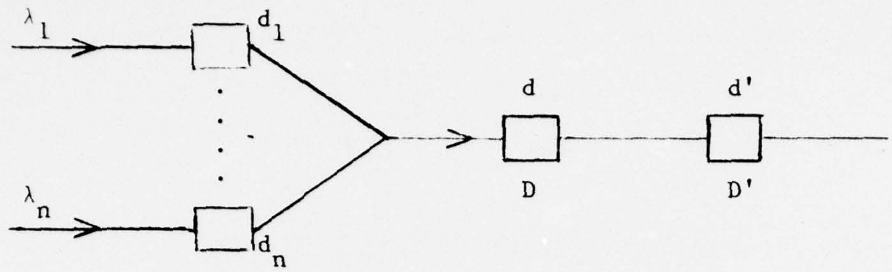

Figure 2

Theorem 4: For $d' \geq d_i$, $i=1,\ldots,n$, the total mean waiting time in the system of Figure 2 is given by

$$\frac{\lambda(\bar{d})^2}{2(1-\lambda\mu\bar{d})}(\mu^2+\sigma^2), \text{ where } \bar{d}=\max(d,d'), \text{ and } \lambda\mu\bar{d}<1,$$

for all values of d.

Proof: If $d \geq d_i$, $i=1,\ldots,n$, then we have shown that the output from D is identical to that of an M/D/1 queue and the result follows immediately.

For $d < d_i$ for some i then $\bar{d} = d'$. It is clear that all arrivals in a particular busy period at D must be served in the same busy period at D'. Note that the delay arguments in Avi-Itzhak apply regardless of the form of the input, so the total waiting time in the system $D \to D' \to$ satisfies:

$$W_{n+1} = (W_n + kd' - t_n)^+ \text{, where } kd' \text{ is the length} \tag{3}$$

of the n-th message, and t_n is the time between the arrival of n-th and (n+1)th messages at D. Now consider the system with the order of the servers D and D' interchanged.

The waiting time in the $D' \to D$ system is still governed by (3), but since $d' \geq d_i$, the results of Theorems 1 and 3 may be applied. Thus the mean waiting time in the $D \to D'$ system is

$$\frac{(\lambda\bar{d})^2}{2(1-\lambda\mu\bar{d})}(\mu^2+\sigma^2) - \sum_{i=1}^{n}\frac{(\lambda_i d_i)^2}{2\lambda(1-\lambda_i\mu_i d_i)}(\mu_i^2+\sigma_i^2)$$

and the output from D' is identical to that from M/D'/1 group arrival queue.

The results of Theorem 4 can obviously be extended to any series system.

Theorem 4 suggests a conjecture about delays when $d_i > d$. Note that reducing any d_i increases the mean wait at D. An intuitive explanation for this is: reducing d_i increases the variance of the ith input stream to D, in most queues the mean waiting time increases with the variance of the input stream. We conjecture that this holds true when some of the d_i are greater than d. Thus if $d_1, \ldots, d_r > d$, $d_{r+1}, \ldots, d_n \le d$, at least for single-packet messages an upper bound for the mean wait at D is

$$\frac{\lambda d^2}{2(1-\lambda d)} - \sum_{i=1}^{r} \frac{(\lambda_i d)^2}{2\lambda(1-\lambda_i d)} - \sum_{i=r+1}^{n} \frac{(\lambda_i d_i)^2}{2\lambda(1-\lambda_i d_i)} . \qquad (4)$$

Clearly such a bound will be lower than that obtained by assuming the input to D is a Poisson process. It takes zero or infinite values at all appropriate points and gives the exact value of the mean wait on the boundary of regions where Theorem 3 applies. In all the examples examined in [3], (4) forms a tight upper bound for the mean waiting time.

It would appear that the use of results for M/D/1 queues gives upper bounds for mean waiting times, and further that by the use of Theorem 4 it should be possible to extend these bounds through many networks.

The Mean Packet Waiting Time.

From Theorem 3 the total mean waiting time of the first packet in each message is

$$\overline{W} = \frac{\lambda d^2}{2(1-\lambda\mu d)} (\mu^2 + \sigma^2).$$

The mean waiting time of the k-th packet in a message is $\overline{W} + kd$. Since

the distribution of the number of packets in a message is known the average packet waiting time in the system, W_p, can be found.

Theorem 5: For $d_i \leq d$, $i=1,\ldots,n, \lambda\mu d<1$.

$$W_p = \frac{d}{2}\left(\frac{\mu^2+\sigma^2}{\mu(1-\lambda\mu d)} - 1\right)$$

Proof: The probability function for the number of packets in a message is $\sum_{i=1}^{n} \frac{\lambda_i}{\lambda} b(i,j) = B(j)$. Since all packets in a message are served consecutively at D, the mean wait of an arbitrary packet in the message is

$$W_p = \overline{W} + \frac{\sum_{j=1}^{\infty} A_j(j-1)d}{\sum_{j=1}^{\infty} A_j}, \quad \text{where } A_j = \sum_{i=j}^{\infty} B(i).$$

Now

$$W_p = \overline{W} + \left(\frac{\sum_{j=1}^{\infty} \frac{1}{2} j(j+1)B(j) - \sum_{j=1}^{\infty} jB(j)}{\sum_{j=1}^{\infty} jB(j)}\right)d,$$

$$= \overline{W} + \left(\frac{\mu^2+\sigma^2-\mu}{2\mu}\right)d,$$

from which the result follows.

The mean waiting time of any packet at D will be

$$\frac{d}{2}\left(\frac{\mu^2+\sigma^2}{\mu(1-\lambda\mu d)}\right) - 1 \quad - \sum_{i=1}^{n} \frac{\lambda_i d_i}{2\lambda}\left(\frac{\mu_i^2+\sigma_i^2}{\mu_i(1-\lambda_i\mu_i d_i)} - 1\right) \qquad (5)$$

From Lemma 1, (5) gives the mean packet waiting time at D under any order of service. In particular it applies when packets are served in order of arrival at D, rather than in order or messages.

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